A new model for Fluidics Transmission Lines

Transmission lines in fluidic systems can be required to handle pneumatic signals with frequency components to as high as 1,000 cps. The simplified line model used in pneumatic process control is grossly inaccurate at such high frequencies. The author presents an accurate model, together with nomographs for speedy determination of gain-vs.-frequency curves.

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The introduction of fluidic devices into the control system designer's tool kit created a problem at the system level. While the characteristics of the fluidic devices themselves were known fairly well, virtually nothing was known of the characteristics of the pneumatic transmission lines used to connect them. Fluidic control systems were designed empirically and the final design, arrived at by trial-and-error, was seldom the optimum one. The need for a simple and sufficiently accurate model of pneumatic transmission lines as used in fluidic control systems is clear.

Process control model works at low frequencies . . .

In fluidic applications, the designer must consider a variety of conditions: transmitted signal frequencies range from less than 1 cps to more than 1,000 cps; line lengths range from inches to about 20 ft; mean pressures range from atmospheric to more than 50 psig; and the size of the pressure signal is often an appreciable fraction of the mean pressure.

Early studies of pneumatic transmission lines concentrated in two areas—conventional pneumatic process control systems and organ pipes or sound tubes. The former concerns very long lines (greater than 50 ft) transmitting low frequency (less than 10 cps) signals. The latter concerns very small signals of very high frequency (greater than 200 cps). Thus, both areas of investigation cover only a limited range of the problems that can arise in fluidic system applications. Other theoretical analyses have been proposed (Ref. 1 and 2), but are not generally used by practicing engineers because of real or imagined mathematical complexity.

Probably the most desirable model of a pneumatic line is its electrical analog (see Box). The model most commonly used for process control work is based on this analog (Ref. 3), and it predicts that the transmission line parameters are independent of frequency. The line parameters—resistance R, inductance L, conductance G, and capacitance C—are expressed in units per in. of line, since they are distributed parameters. The process control model expresses these parameters as

$$R = \frac{8\pi\mu}{A^2} \qquad G = 0$$
$$L = \frac{\rho}{A} \qquad C = \frac{A}{\gamma P} \tag{1}$$

where μ is the fluid viscosity (psi-sec), A the line cross-sectional area (in.^{2}), ρ the fluid density (lb-sec^2/in.^{4}), γ the ratio of specific heats (dimensionless), and P the mean line pressure (psi). The fluid characteristics μ, ρ, and γ are functions of temperature and pressure, but for a given set of operating conditions the line parameters are constants. This analysis assumes that the line does not leak.

. . . but not for high frequencies

More thorough studies of pneumatic transmission line dynamics took into account thermal as well as viscous boundary layer effects (Ref. 1 and 2), and demonstrated that the line really has two distinct transmission regimes. At low frequencies, the line parameters are independent of frequency, as described above. At high frequencies, the line parameters are functions of frequency. A characteristic frequency f_v serves as a demarcation line between the two regimes. This char-

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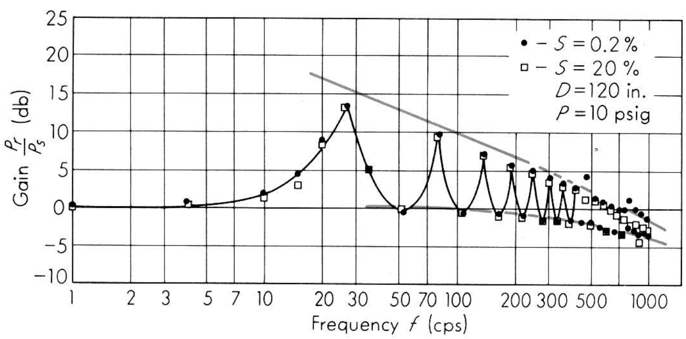


FIG. 1. The experimentally determined transmission line gain curve for two different signal levels falls accurately within the gain envelopes (colored lines) predicted by the high-frequency model of Equations 2. Note the minor shifts in the gain curve caused by a change of signal level from 0.2 percent (circles) to 20 percent (squares). Note also that the agreement between the test data and the calculated gain envelopes starts to deteriorate at a frequency of about 400 cps, or 15 times the fundamental frequency.

acteristic freqeuncy is defined by the expression

$$f_v = \frac{4\nu}{A}$$

where the kinematic viscosity $\nu = \mu/\rho$, and is expressed in in.^{2} per sec.

Nichols' equations defining the line parameters in the high frequency regime may be greatly simplified to the following expressions

$$R = \pi L \sqrt{ff_v} \qquad G = \frac{(\gamma - 1)}{\sigma} \pi C \sqrt{ff_v}$$
$$L = \frac{\rho}{A} \qquad C = \frac{A}{\gamma P} \tag{2}$$

provided that the signal frequency f is much greater than the characteristic frequency f_v (Ref. 4).

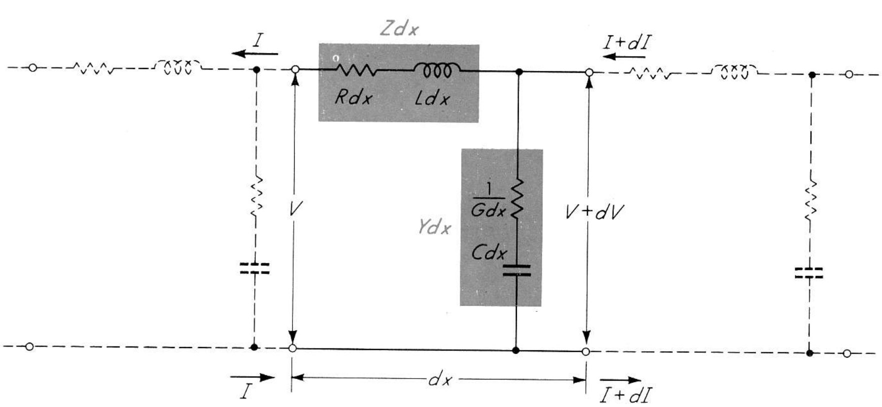
The dimensionless Prandtl number σ is defined as

$$\sigma^2 = \frac{c_p \mu}{k}$$

where the heat capacity at constant pressure c_p, the absolute viscosity μ, and the thermal conductivity k are all expressed in consistent units.

The unit inductance and capacitance in this high-frequency model are the same as in the constant-resistance model used for process control. But note that here a conductance exists without physical leakage, and both the unit resistance and conductance are functions of the signal frequency f.

THE ELECTRIC-PNEUMATIC ANALOGY



A pneumatic transmission line, like an electric transmission line, is a distributed parameter system, and the well-known electrical model also provides a model for this line. Both systems may be represented by similar differential equations and, based on this model, the well-developed theory of electrical transmission may be applied directly to pneumatic lines.

In the pneumatic model, pressure P is analogous to voltage V, and volumetric flow rate Q is analogous to current I. The steady state resistance of the line is the amount of driving force or potential required per unit of flow—V/I or P/Q. Expressing P in psi and Q in cu in. per sec or cis, the pneumatic units of resistance are psi/cis.

But the pneumatic line, like the electric line, has transient effects. The line has capacitance because pressure changes throughout its length cause air to flow. And it has inductance because any flow of air tends to change the pressure distribution. The dynamic model of each line section, see figure, consists of a series impedance Z (psi/cis) and a shunt admittance Y (cis/psi). Resistance R is the real part of Z, and inductance L is $1/\omega$ times the imaginary part. Conductance G is the real part of Y and capacitance C is $1/\omega$ times the imaginary part.

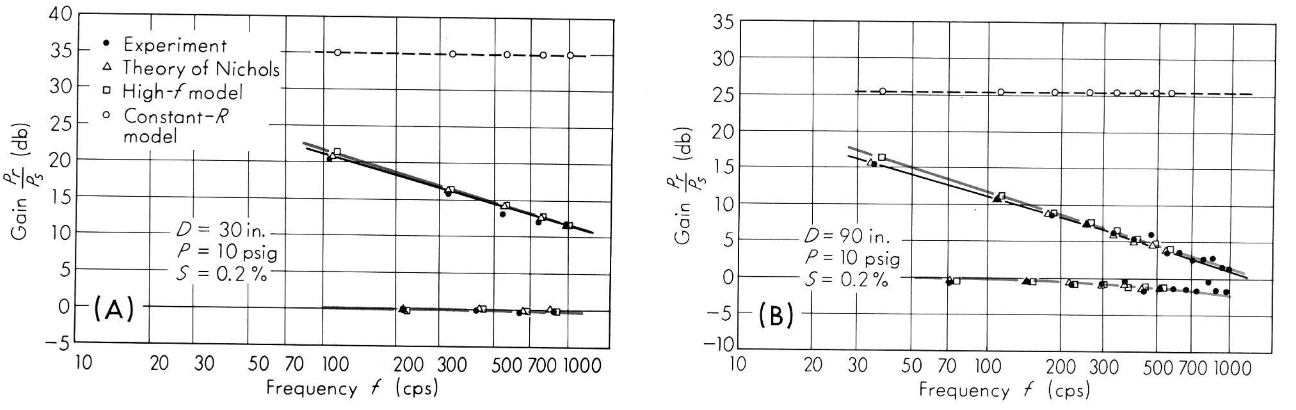
Physically, the line resistance is due to viscous effects; inductance results from the fluid's inertia; capacitance represents the energy-storing ability of the fluid; and conductance is due to thermal effects or actual flow leakage.

The units of pneumatic inductance and capacitance are, by analogy with the electrical equivalent:

$$L = \left[\frac{V}{dI/dt}\right] = \frac{P}{dQ/dt} = \frac{psi\text{-}sec}{cis}$$

$$C = \left[\frac{I}{dV/dt}\right] = \frac{Q}{dP/dt} \quad \frac{cis\text{-}sec}{psi}$$

Because the line is a distributed network, all line parameters in the text equations are expressed in unit terms—per in. of line. For example, unit inductance is expressed as psi-sec/cis-in.



HOW 3 MODELS COMPARE

FIG. 2. Resonant frequency gains calculated from the constant-resistance model, the high-frequency model, and Nichols' model are plotted here, together with experimentally determined values for two different line lengths. Over most of the frequency range, the high-frequency model and Nichols' more exact method agree very well with the test data, but the constant-resistance model does not.

The gain envelope method

The dynamic characteristics of a pneumatic transmission line are commonly presented as frequency response curves showing amplitude and phase shift as a function of frequency. However, in the high-frequency regime, characterized by

$$RG << 4\pi^2 f^2 LC$$

transmission line losses are low, and all low-loss lines have almost identical gain curves. The gain curves for such lines differ only in the magnitude of their maxima and minima and in the frequencies at which these singularities occur. This fact permits a simple yet accurate dynamic analysis for high frequencies, by means of gain envelopes.

The gain envelopes are simply the loci of the maxima and minima on the amplitude frequency response curve. For lines having a blocked termination, the gain envelopes are defined by (Ref. 4)

$$Gain_{\max} = (\sinh \alpha D)^{-1}$$
$$Gain_{\min} = (\cosh \alpha D)^{-1} \qquad (3)$$

where α is the unit attenuation (neper per in.) and D is the total line length (in.). The unit attenuation is itself a function of frequency. For high-frequency, low-loss lines it is given by

$$\alpha = \frac{LG + RC}{2\sqrt{LC}} = \left[\frac{(\gamma - 1)}{\sigma} + 1\right]\sqrt{\frac{\pi^2 \mu f}{A\gamma P}} \qquad (4)$$

For a given low-loss line, this reduces to a direct proportionality between α and the square root of frequency.

The gain envelopes for a given transmission line can be plotted from Equations 2, 3, and 4, evaluated at a number of frequencies arbitrarily selected to cover the range of interest. Then, to obtain the gain curve itself, the points of maximum and minimum gain are plotted on the gain envelopes and connected with smooth curves. The gain maxima and minima occur at those frequencies where the line is resonant. For the two models discussed in this article (Equations 1 and 2), the resonant frequencies are given by

$$f_n = \frac{na}{4D} \qquad (5)$$

where the adiabatic sonic velocity $a = (\gamma P/\rho)^{1/2}$, is expressed in in. per sec, and n takes on successive integer values. For a blocked line, the gain is maximum for odd resonant frequencies ($n = 1, 3, 5 \ldots$) and minimum for even resonant frequencies ($n = 2, 4, 6, \ldots$). The resonant frequencies determined from Equation 5 are plotted along the previously determined gain envelopes to locate the points of inflection on the gain curve for the given transmission line.

A series of experiments was run on blocked ¼-in. OD pneumatic transmission lines to evaluate the accuracy of the high-frequency model. A blocked line was chosen in these tests as a starting point for later more realistic studies. The blocked line represents the simplest form of the transmission line problem, and allows verification of the high-frequency model presented here and the gain envelope technique, without involving the more difficult problem of modeling the termination.

In the tests, gain values at controlled frequencies were determined from measurements of signal pressures at the source P_s and at the receiver P_r. The gain curves determined by test fitted nicely within the gain envelopes predicted by the high-frequency model (Equations 2). Figure 1 shows this fit at two different signal levels. Signal level S is expressed as the ratio of the ac component of pressure (variation around P) to the dc or average pressure P. Even at the high signal level of 20 percent, gain envelopes based on the high-

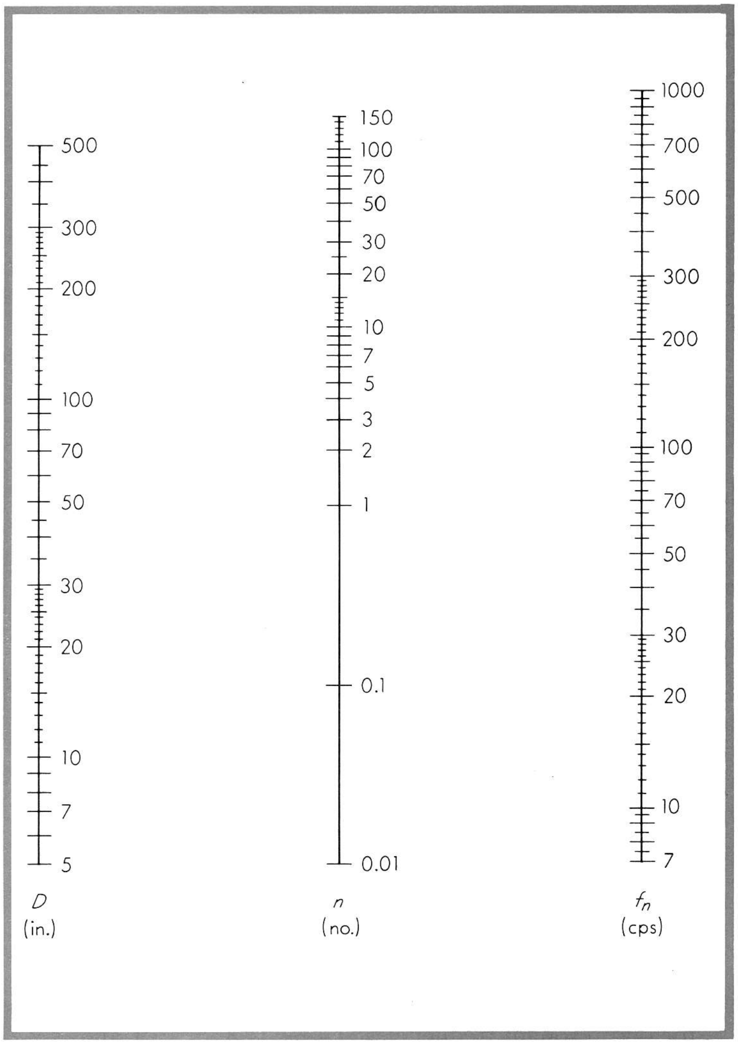


FIG. 3. **RESONANT FREQUENCY NOMOGRAPH**

frequency model fit the experimental results.

Further results for different line lengths, Figure 2, also show close agreement between the high-frequency model and the test data. Resonant frequencies calculated by the exact method of Nichols (Ref. 2) are also plotted on Figure 2, as well as values based on the constant-resistance model (Equations 1). Nichols' exact solution agrees well with the test data and the high-frequency model, but the constant-resistance model fails completely at these high frequencies.

In all 29 tests, the high frequency model predicted resonant gains within 2 db of the experimental values, and predicted the resonant frequencies within 10 percent. Usually, the agreement was much better than these limits.

Which model to use

Based on the results shown in Figures 1 and 2, the high-frequency model (Equations 2) should be used whenever the signal frequency is greater than the characteristic frequency of the line by a factor of 5 or more. For ¼-in. OD lines, the characteristic frequency is on the order of 1 cps, so the high-frequency model applies to practically the entire range of signals encountered in fluidics (1 to 1,000 cps). It does not apply at the lower signal frequencies when the lines are very small (⅛-in. OD or less) or when the lines are long (20 ft or more). In these cases, or whenever an exact solution is required, Nichols' method should be used. The only disadvantage of Nichols' method is its increased complexity and the resulting time consumption and lack of manipulative ease.

While the constant-resistance model (Equations 1) has been used successfully in the study of process control transmission lines, it should never be used when the signal frequencies are greater than the characteristic frequency of the line.

When using the gain envelope method with any model, the higher resonant frequencies (for $n > 15$) will be in large error. This is so because any error in the fundamental frequency ($n = 1$) is also multiplied by n. However, at these higher frequencies there is usually very little difference between the maximum and minimum gains, and the converging gain envelopes may be considered as limiting or bracketing values of the actual gain.

The gain envelope nomograph

The main advantage of the high-frequency model and the gain envelope technique is their ease of application. For a given line, the describing parameters are simply constants or constants times the square root of the signal frequency. The simplicity of the equations defining these parameters makes nomographic solutions possible. Nomographs for the resonant frequencies and the attenuation of cylindrical, pneumatic transmission lines, Figures 3 and 4, can be used to determine the gain curve of practically any pneumatic line having a blocked termination. The nomographs are based on air at 80 deg F ($\sigma^2 = 0.704$, $\gamma = 1.4$), but are valid within a few db for ambient temperatures from 30 deg F to 130 deg F. The step-by-step procedure is as follows:

1. Determine the length D of the transmission line in in., its diameter d in in., the mean pressure P in psia, and the range of signal frequencies f in cps.
2. On Figure 3, connect D and f_{max} to determine n_{max}. If $n_{max} \ll 1$, resonance does not occur and the line may be represented by a single equivalent circuit (one section ladder network). If $n_{max} \geqq 1$, proceed to the next step.
3. On Figure 4, connect d and P to determine f_v. If $f_v > f_{min}$, use a low frequency method (constant resistance model) for signal frequencies lower than f_v and proceed to the next step for frequencies higher than f_v. If $f_{min} > f_v$, proceed to the next step.
4. On Figure 4, connect f_v and f to determine α for several arbitrary values of f selected to cover the range of interest.

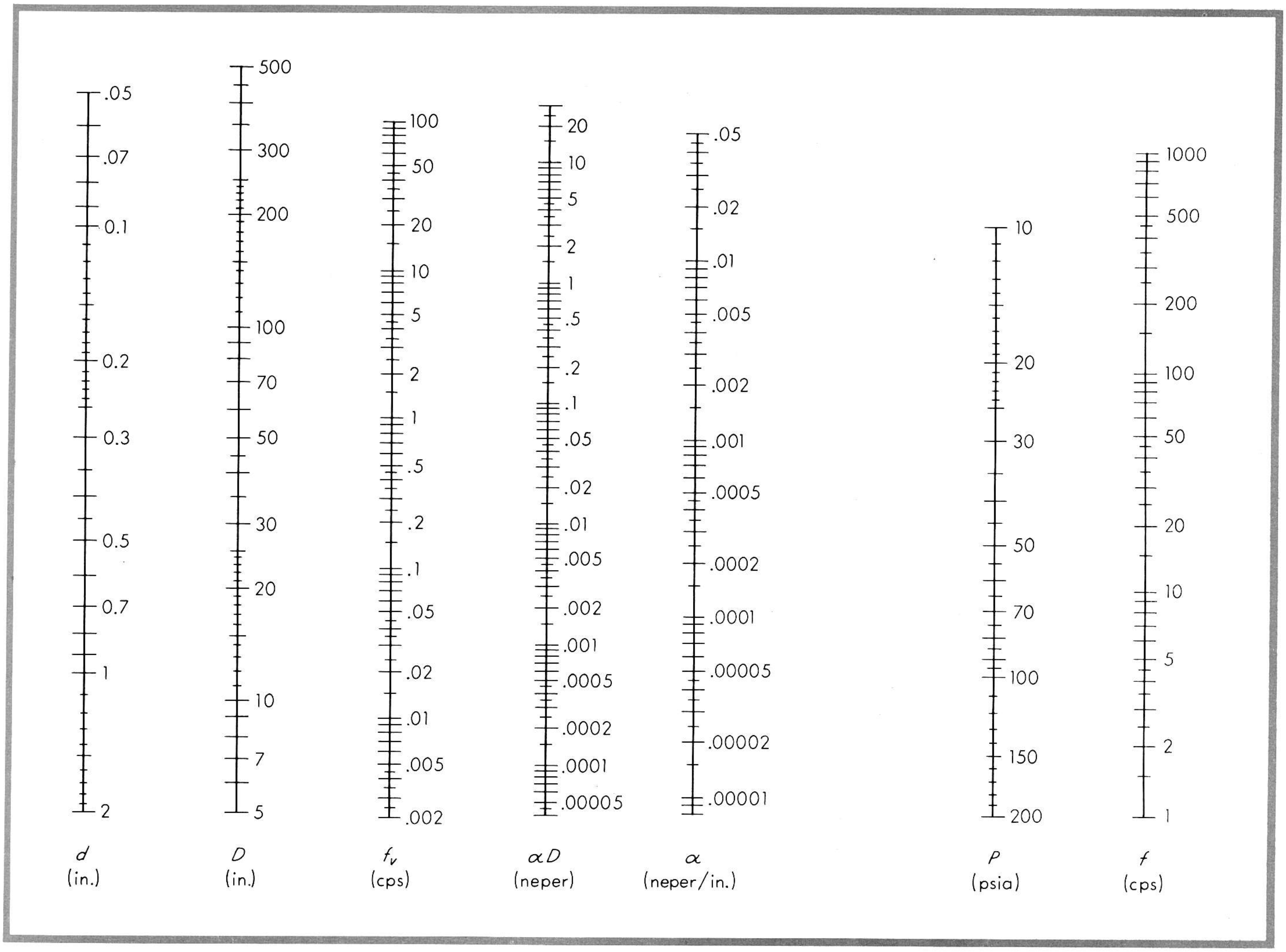


FIG. 4. **ATTENUATION NOMOGRAPH**

5. On Figure 4, connect D and the values of α obtained in Step 4 to determine αD for the selected frequencies.
6. Using a hyperbolic function table, determine the maximum and minimum gains at the selected frequencies from Equations 3. Convert the gains to decibels [db $= 20 \log_{10}$ (Gain)].
7. Plot the gains in db versus the corresponding frequency on semilog paper. Connect the maximum gain points to each other and the minimum gain points to each other, thereby forming the gain envelopes.
8. On Figure 3, connect D and n to determine the resonant frequencies f_n for all integral values of n from 1 to 15. For odd values of n, plot the corresponding f_n on the maximum gain curve. For even values of n, plot the corresponding f_n on the minimum gain curve.
9. Connect the plotted resonance points together, thereby forming the gain curve.

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